Erosion as a driving mechanism of intracontinental mountain growth

J. P. Avouac
Laboratoire de Géophysique, Bruyères-Le-Châtel, France

E. B. Burov
Laboratoire de Gravimétrie et Géodynamique, Institut de Physique du Globe de Paris

Abstract. In nature, mountains can grow and remain as localized tectonic features over long periods of time (> 10 m.y.). By contrast, according to current knowledge of lithospheric rheology and neglecting surface processes, any intracontinental range with a width that exceeds that which can be supported by the strength of the lithosphere should collapse within a few tens of millions of years. For example, assuming a quartz-dominated crustal rheology, the relief of a range initially 3 km high and 300–400 km wide is reduced by half in about 15 m.y. as a result of lateral spreading of its crustal root. We suggest that surface processes might actually prevent such a “subsurface collapse.” Removal of material from topographic heights and deposition in the foreland oppose spreading of the crustal root and could eventually drive a net influx of material toward the orogeny. We performed a set of numerical experiments in order to validate this hypothesis. A section of a lithosphere, with a brittle-elasto-ductile rheology, initially loaded by a mountain range is submitted to horizontal shortening and to surface processes. If erosion is intense, material is removed more rapidly than it can be supplied by crustal thickening below the range, and the topography is rapidly smoothed. For example, a feature 3 km high and 300–400 km wide is halved in height in about 15 m.y. for an erosion coefficient $k = 10^3$ m$^2$/yr (the erosion rate is of the order of a few 0.1 mm/yr). This regime might be called “erosional collapse.” If erosion is not active enough, the crustal root spreads out laterally and “subsurface collapse” occurs. In the third intermediate regime, removal of the material by erosion is dynamically compensated by isostatic rebound and inward flow in the lower crust so that the range can grow. In this “mountain growth” regime the range evolves toward a characteristic graded shape that primarily depends on the erosion law. The erosion rate may be high (e.g., 0.5–1.1 mm/yr), close to the rate of tectonic uplift (e.g., 0.7–1.1 mm/yr), and few times higher than the rate of topographic uplift (0.15–0.2 mm/yr). These experiments show that surface processes can favor localized crustal shortening and participate in the development of an intracontinental mountain. Surface processes must therefore be taken into account in the interpretation and modeling of long-term deformation of continental lithosphere. Conversely, the mechanical response of the lithosphere must be accounted for when large-scale topographic features are interpreted and modeled in terms of geomorphic processes.

Introduction

Intracontinental mountain ranges are generally interpreted to have resulted from localized crustal thickening in response to horizontal shortening. The growth and maintenance of such topographic features within continents might be taken to indicate that the strength of the crust exceeds the deviatoric stresses associated with slopes of the topography and of the Moho. Yet, laboratory experiments indicate that at the pressure and temperature conditions of the lower crust, most crustal rocks should flow easily [e.g., Brace and Kohlstedt, 1980; Wang et al., 1994]. Irregularities of the topography and of the Moho should therefore be relaxed by viscoplastic flow in the ductile lower crust and decay with time [Kuszniir and Matthews, 1988; Gratton, 1989; Bird, 1991]. Consider for example the Tien Shan, which is, except for the Himalayas, the largest and most active intracontinental range in the world (Figure 1). The range is 300–400 km wide in its central portion, with a mean elevation of about 3500 m, in a zone of relatively thick and tectonized crust (Moho depths from 50 to 70 km) [e.g., Avouac et al., 1993]. A simple dimensional analysis [Gratton, 1989] as well as numerical simulations [Bird, 1991] show that the topography of such a range should be reduced by half in a few million years or at most tens of millions of years, depending on the assumed rheology of the lower crust. Only the short topographic wavelengths, typically less than a few tens of kilometers, that can be supported by the strength of the upper crust would be maintained over geological periods of time. In addition, surface processes might be thought to contribute to an even more rapid smoothing of the topography. Altogether, in the absence of strain localization processes a portion of a continental crust submitted to horizontal shortening is expected to thicken homogeneously so that no mountain should
form. The growth and maintenance of an intracontinental mountain range over long periods of time must therefore involve dynamical processes allowing for long-term localization of lithospheric strain below the mountain. Several mechanisms have been advocated. Intrinsic strain-softening properties of rocks could sustain localized thrust faulting at the crustal scale. Alternatively, a range could result from shear stresses at base of the crust induced by lithospheric underthrusting or by some mantle dynamics [e.g., Beaumont et al., 1994; Ellis et al., 1995]. Such a mechanism may be suggested for mountains associated with subduction zones or with hotspots [Vogt, 1991] but seems inappropriate to explain most intracontinental mountains. In the case of the Tien Shan, a particular mantle dynamics has been inferred from gravity modeling [Burov et al., 1990] and seismic anisotropy [Makeyeva, 1992; Roecker et al., 1993], but we contend that it might not be the key factor. Our point is that coupling between surface processes and flow in the lower crust could provide an alternative and general explanation.

In this paper we first show that surface and tectonic processes are not independent processes and that they can interact. We suggest in particular that advection of material at the Earth's surface and horizontal flow in the crust might be coupled and that this coupling could permit mountain growth in response to horizontal shortening. This mechanism is then validated and investigated on the basis of numerical experiments in which the evolution of an already mature mountain range is modeled. The model accounts for the rheological layering of the lithosphere and for surface processes. We find that depending of the erosion rate compared to horizontal shortening, flow in the lower crust can be "outward" (from under the high topography) or "inward" (toward the crustal root of a high topography). When inward flow occurs, a mountain range can grow and no other mechanism is required to explain localized uplift. Some implications about the role of climate on continental tectonics and on the geomorphology of mountain ranges are then derived.

Interplays Between Surface and Tectonic Processes

Tectonic Forcing on Surface Processes

Topographic contrasts are required to set up erosion and sedimentation processes. Tectonics is therefore a forcing factor of surface processes. Following Ahnert [1970] and Pinet and Souriau [1988], Summerfield and Hulton [1994] have recently compiled rates of denudation at the scale of major river basins. These studies indicate that denudation is primarily influenced by basin topography so that rates of denudation appear to be systematically high in areas of active tectonic uplift. Common
values of mean denudation rates in such areas would be of the order of a few 0.1 mm/yr to about 1 mm/yr at the scale of large drainage basins. Such rates are generally consistent with estimates derived from balancing sediment volumes over geological periods of time [Leeder, 1991; Summerfield and Hutton, 1994]. Thermochronologic studies indicate, however, local values as great as 1 mm/yr (see Leeder [1991] and Molnar and England [1990] for critical reviews). The discrepancy between local and basin averaged estimates is due to the fact that tectonic uplift is probably distributed in brief pulses over localized domains within a drainage basin [Copeland and Harrison, 1990]. In the absence of any tectonic feedback, common values of denudation rates should lead to the disappearance of a major mountain belt like the Tien Shan in a few million years. Pinet and Souriau [1988] demonstrated that denudation leads to an exponential decay of the topography of a range with a characteristic time constant of the order of 2.5 m.y.

**Coupling Between Denudation and Tectonic Uplift Due to Isostasy**

Isostasy provides an immediate mechanism that should link subsurface and surface processes. Redistribution of surface loads by erosion and sedimentation must induce tectonic deformation to maintain isostatic balance. Vertical uplift is expected to partly compensate unloading in the area subjected to denudation, while subsidence should occur in response to loading by sedimentation. This feedback mechanism may lead to some coupling between denudation and tectonic uplift [e.g., Ahnert, 1970]. A first consequence is that the time needed to erode a topographic relief must take into account removal of the topographic relief and of the crustal root. If local isostasy is assumed and if horizontal strains are neglected, denudation is dynamically compensated by uplift, and the characteristic time of decay of the topography would then be of the order of 10 m.y. [Leeder, 1991]. It has been argued that some kind of positive feedback may then arise [Molnar and England, 1990; Masek et al., 1994]. If the slopes of valleys steepen during river incision, isostatic readjustment following denudation in a mountain range may result in a net uplift of the higher summits in spite of the average lowering of reliefs. Erosion might thus induce some uplift of topographic summits, leading in turn to enhanced erosion. A rapid uplift of the Himalayan belt during the last few million years may have resulted from this process [Burbank, 1992]. Note, however, that while the peaks might reach higher elevations following isostatic adjustment, the net effect of erosion in that mode is crustal thinning. Thus this effect does not seem to be a fundamental ingredient in mountain-building processes.

**Coupling Between Surface Processes and Horizontal Strains**

As mentioned in the introduction, small lateral variations of the crustal thickness should drive horizontal flow in the lower crust. Some studies have already pointed out the importance of such a process in continental tectonics [e.g., Lobkowsky, 1988; Lobkowsky and Kerchman, 1991]. For example, Kruse et al. [1991] have shown that horizontal flow in the lower crust has regulated isostatic equilibrium during extension in the Basin and Range. The lower crust would have been extruded from under the high topography during that process. Following Westaway [1994], this sense of flow is called “outward.” On the other hand, Gregory and Chase [1994] inferred “inward” flow, toward the crustal root, during the Laramide orogeny of the Frontal Range, Colorado. The characteristic time associated with flow in the lower crust, induced by the topography of a range of a few thousand meters high a few hundred kilometers wide, is of the order of a few million years. The characteristic times of erosional decay of the topography of a range and of lateral collapse of a crustal root are thus of the same order of magnitude. Since both processes are driven by topographic slopes, some coupling may arise. Although it is not often pointed out, it has long been recognized that this kind of process might play a major role in elevation changes within continents (see Westaway [1994] for an review of historical development of these ideas). Westaway [1994] made a case for such a coupling, with inward flow, in the context of extensional tectonics in western Turkey. He proposed that sediment loading in the sedimentary basins would have driven flow toward the uplifted area. This kind of process was first modeled by King and Ellis [1990], who modeled crustal extension using a thin elastic plate (upper crust) overlaying an inviscid fluid (lower crust).

We propose that this kind of coupling might also appear in a compressional context. Let us consider a portion of a lithosphere, loaded with some initial range in regional isostatic balance and submitted to horizontal compression (Figure 2a). Because a thicker crust and bending stresses tend to reduce the strength of the lithosphere [Burov and Diament, 1995; Ranalli, 1994], the lithosphere is weaker beneath the range. At the same time, the stresses resulting from the slopes of the topography and of the Moho should drive horizontal flow in the lower crust beneath the high topography. In the absence of horizontal shortening and erosion, the lower crust below the range would be extruded laterally, as discussed by Bird [1991] (Figure 2a). Our point is that if erosion takes place, a regime may be established in which horizontal shortening could be preferentially accommodated by crustal thickening in the area below the range (Figure 2b). Surface processes remove material from the range and feed the adjacent flexural basins, inducing isostatic imbalance. This imbalance produces in turn a temporary excess of normal stress below the foreland basins and a deficit below the range, favoring flow in the lower crust toward the crustal root. Ultimately, this coupled regime might lead to some dynamic equilibrium in which the amount of material removed by erosion would balance the material supplied to the range by subsurface deformation.

**Modeling: Principles and Numerical Implementation**

We developed a model in order to validate the coupled regime described in the previous section. For this purpose the model has to account for (1) surface processes, (2) the effect of topographic loads and of variations of crustal thickness on the mechanical behavior of the lithosphere, and (3) ductile flow in the crust. Therefore we have had to consider a depth and strain dependent rheology of the lithosphere. In this section we propose some approximations and a numerical procedure to account for these basic properties. The resulting model allows computation of the deformation of a two-dimensional section of continental lithosphere subject to horizontal compression and to surface processes.

**Surface Processes**

We looked for a simple two-dimensional law that could simulate erosion and sedimentation at the scale of a mountain range. The evolution of a landscape results from the combina-
Figure 2. Velocity field in a deforming section of crust loaded with some range topography and submitted to horizontal shortening. We assume that the thick crust beneath the range is relatively weak and mechanically decoupled from the upper mantle. (a) In the absence of surface processes the crustal root below the range is expected to spread out laterally ("outward" flow). (b) Denudation in highlands and sedimentation in lowlands may drive "inward" flow in the lower crust: denudation, which removes material from steep and elevated topography, results in uplift due to isostatic rebound. Sedimentation, which transports eroded material to the lowlands, produces increased subsidence due to the increased load. Unloading beneath the range and loading beneath the piedmonts may thus force the lower crust to flow toward the mountain root. As a result, topographic uplift may be localized below the range, and the range can grow rather than spread laterally.

\[
\frac{\partial h}{\partial t} = -k \frac{\partial q_e}{\partial x}. \tag{1}
\]

With constant \( k \), equations (1) and (2) lead to the linear diffusion equation

\[
\frac{\partial h}{\partial t} = k \frac{\partial^2 h}{\partial x^2}. \tag{2}
\]

This model of surface processes holds only for particular conditions. The regolith must form more rapidly than it is removed by surface transport, and slopes must not exceed the frictional angle of the material. Even for scarps formed in loose alluvium some complications arise when high scarps are considered. Scarps with height typically in excess of about 10 m in arid climatic zones tend to have systematically sharper curvatures at crest than at base [e.g., Andrews and Bucknam, 1987]. At the larger scale, hillslope and stream processes interact and the sediment transport then depends nonlinearly on the slope and on other factors such as the slope gradient, the area of weathering processes that prepare solid rock for erosion, and transportation by hillslope and stream processes (see Carson and Kirkby [1972] for a review). Although many factors, depending on the lithologies and on climate [e.g., Fournier, 1960], may control this evolution, quite simple mathematical models describing the geometrical evolution of the morphology at the small scale have been proposed and tested successfully [e.g., Chorley et al., 1984; Kirkby, 1986]. For example, the two-dimensional (2-D) evolution of a scarplike landform can be modeled assuming that the rate of downslope transport of debris, \( q_e \), is proportional to the local slope [Culling, 1960; Hanks et al., 1984; Avouac, 1993]:

\[
q_e = -k \frac{\partial h}{\partial x}, \tag{1}
\]

where \( k \) is the mass diffusivity coefficient, expressed in units of area per time (e.g., m²/yr). In the assumption of mass conservation along a 2-D section and no tectonic deformation, \( h \) must obey
drained above the point, and the distance from the water divide, so that the simple 2-D linear diffusion does not apply in general [e.g., Gossman, 1976]. In spite of these limitations, we have chosen to stick to a linear diffusion law to model erosion in the upland. This model does not accurately mimic the spatial distribution of denudation in the mountain range, but it leads to a sediment yield at the mountain front that is roughly proportional to the mean elevation of the basin relative to that point (a rough approximation to the sediment yield resulting from a change of elevation $h$ over a horizontal distance $d$ is $k \times h/d$) and therefore accounts for the apparent correlation between elevation and denudation rates observed by Ahnert [1970], Pinet and Souriau [1988], and Summerfield and Hulton [1994]. We did not apply the diffusion model to the whole system, however. We felt that we should take into account the major discontinuity in surface processes that occurs at the mountain front. As a river emerges into the adjacent basin, its gradient is sharply reduced and deposition occurs. The streams shift from side to side and build up alluvial fans that tend to form a broad gently sloping pediment at the base of the mountain range. In addition, a lateral drainage often develops along the foothills of mountain ranges (for example, the Gange along the Himalayas, the Parana along the Andes, or the Tarim along the Tien Shan). Altogether the formation of the pediment and lateral drainage tend to maintain gentle slopes in the foreland. There is therefore a sharp contrast between river incision that maintains a rugged topography with steep slopes in the mountain range and widespread deposition of alluvium in the foreland. This discontinuity of processes must be considered to model the sharp break in slope at the mountain front that is generally observed on topographic profiles across mountain land. This discontinuity of processes must be considered to model the sharp break in slope at the mountain front that is generally observed on topographic profiles across mountain land.

We considered values for $k$ varying between $10^3$ and $10^4$ m$^2$/yr that yield denudation rates of the order of a few tenths of a millimeter per year to 1 mm/yr for a 200- to 400-km-wide range with a few thousand meters of relief.

In order to test the sensitivity of our model on the assumed erosion law we also considered nonlinear erosion laws of the form

$$\frac{\partial h}{\partial t} = k^*(x, h, \frac{\partial h}{\partial x}) \frac{\partial^2 h}{\partial x^2}, \tag{4}$$

where

$$k^*(x, h, \frac{\partial h}{\partial x}) = k(x)\left(\frac{\partial h}{\partial x}\right)^n. \tag{5}$$

[e.g., Gossman, 1976; Andrews and Bucknam, 1987]. We will refer to the cases with $n = 1, 2$ as first- and second-order diffusion, respectively. In these cases we did not introduce the change in regime at the mountain front since the nonlinear effects already tend to form relatively smooth pediments. It should be noted that (4) differs from that obtained assuming a nonlinear diffusion coefficient in (1). In that case conservation of mass would lead to an additional term $(\partial k^*/\partial x)(\partial h/\partial x)$ in (4).

### Structure and Rheology of the Lithosphere

Previous investigations of the interplay between erosion and tectonics have generally been conducted assuming either local isostasy [Ahnert, 1970; Leeder, 1991] or thin plate behavior of the lithosphere [Beaumont, 1981; Flemings and Jordan, 1990; Beaumont et al., 1992; Masek et al., 1994]. Some authors have actually considered the possibility for ductile flow in the lower crust and treated the lower crust as an inviscid fluid overlaid by a thin elastic plate [King et al., 1988; King and Ellis, 1990]. The effects of surface loads and of the variations of crustal thickness on the mechanical behavior of the lithosphere have been neglected. The coupled regime described in the previous section assumes that strain localization below a range results from weakening of the lithosphere due to crustal thickening and bending stresses. In order to account for this process we can treat the lithosphere neither as a one-layer elastic plate with vertically integrated properties overlaying an inviscid asthenosphere nor as a thin viscous sheet [e.g., England and McKenzie, 1983; Vilotte et al., 1982]. We consider the lithological and mechanical rheological layering of the lithosphere (Figure 3).

Three lithological layers are distinguished: the upper crust, the lower crust, and the mantle. Each layer has specific properties: density, mechanical, and thermal constants (see notation section). We assume no compositional changes due to deformation or cooling. The lithological boundary between the upper and lower crust lies at a fixed depth of 20 km. The bottom of the mantle lithosphere is limited by the 1330°C isotherm at a depth of about 250 km.

At small differential stresses the rocks behave elastically. In terms of principal components the relationship between the stress tensor $\sigma$ and the strain tensor $\varepsilon$ can be written

$$\sigma_j = 2\mu\varepsilon_j + \lambda(\varepsilon_1 + \varepsilon_2 + \varepsilon_3) \quad (j = 1, 2, 3). \tag{5}$$

The parameters $\lambda$ and $\mu$ are Lamé’s constants related to Young’s modulus $E$ and Poisson’s ratio $\nu$ as

$$\lambda = \frac{E \nu}{(1 + \nu)(1 - 2\nu)}; \quad \mu = \frac{E}{2(1 + \nu)}. \tag{6}$$

Strain weakening by brittle failure or ductile flow occurs when elastic stresses reach some threshold above which unrecoverable strain grows without increase of stress. The conditions of brittle failure are independent of rock type and temperature but are strongly controlled by pressure [Bryan, 1978]:

$$\sigma_3 = (\sigma_1 - \sigma_3)/3.9 \quad \sigma_1 < 120 \text{ MPa};$$

$$\sigma_3 = (\sigma_1 - \sigma_3)/2.1 - 100 \quad \sigma_1 > 120 \text{ MPa}. \tag{7}$$

This law should be written in terms of effective stresses to account for the effect of pore pressure. In this study we do not consider fluids.

Ductile flow in the lithosphere essentially results from thermally activated dislocation creep [e.g., Kusznir, 1991]:

$$\dot{\varepsilon} = A^* \exp\left(-H^*/RT\right)(\sigma_1 - \sigma_3)\nu. \tag{8}$$

The values of the parameters are given in Table 1. The ratio of the stress to strain rate defines an effective non-Newtonian viscosity:

$$\mu_{eff} = (\sigma_1 - \sigma_3)/2\dot{\varepsilon}$$
Figure 3. Yield strength envelope (equation (9)) of a 400 Ma continental lithosphere with a 50-km-thick crust, submitted to horizontal shortening with $\dot{e}_{\text{xx}} = 5 \times 10^{-15} \text{s}^{-1}$. $\Delta\sigma = \sigma_1 - \sigma_3$ is the differential stress ($\sigma_1$ and $\sigma_3$ are maximum and minimum principal stresses, respectively). As discussed in the text, the strength of the brittle domain is higher when the lithosphere is submitted to horizontal compression than when it is submitted to horizontal extension. In this example, the lower crust has a low temperature of creep activation and thus is mechanically decoupled from mantle. Dashed lines show ductile yield stress obtained for various strain rates between $10^{-17}$ and $10^{-13} \text{s}^{-1}$. The thick line in Figure 3a indicates the deviatoric stresses in the lithosphere when submitted to $1.5 \times 10^{13} \text{ N m horizontal compression}$. The parameters $h_{1bd+}$ and $h_{1bd-}$ correspond to the depths of the brittle-ductile transition in the crust for tensional and compressive regimes, respectively; $h_{2bd+}$ and $h_{2bd-}$ correspond to those in the mantle. The depth distribution of deviatoric stresses is shown in Figure 3b when an additional downward bending moment of $7 \times 10^{17} \text{ N m/m}$ is applied. The elastic cores in the crust and upper mantle are reduced due to flexure of lithosphere. Bending stresses lead to inelastic behavior (flow and brittle failure) in large zones of mantle and crustal lithosphere (e.g., material flows below the point $y_{13}$ in crust and below the point $y_{23}$ in mantle). All other notations are defined in Appendix A.
It results in the yield when submitted to increasing horizontal forces (Figure 3a) or whereas for olivine it should be $600^\circ$ - $700^\circ$C [e.g., Brace and Carter and Tsenn, 1987]. See equation (7). The data sources are Brace and Kohlstedt [1980], Carter and Tsenn [1987], and Kirby and Kronenberg [1987].

Although ductile deformation occurs even under low differential stresses, a ductile yield strength can be defined. If boundary conditions are given in terms of rate of displacement, a average strain rate can be derived. This "basic" strain rate is used to define a stress threshold from (7). Owing to the nonlinearity in (7), if the stress is slightly less than this level, ductile deformation will process very slowly so that most of the imposed deformation will be absorbed elastically. For example, for olivine the strain rate decreases by a factor of 1000 if the stress lies 10–15% below the "threshold" level. On the other hand, the stress level cannot significantly exceed this threshold since it would require a strain rate much higher that the one that is imposed from boundary conditions. This threshold thus defines a ductile yield strength. A temperature about $250^\circ$ - $300^\circ$C must be exceeded for ductile deformation of quartz, whereas for olivine it should be $600^\circ$ - $700^\circ$C [e.g., Brace and Kohlstedt, 1980; Carter and Tsenn, 1987]. It results in the yield stress envelope (YSE) being controlled by the conditions for brittle failure in the shallow crust and upper mantle and by the condition for ductile failure in the deep crust and deep upper mantle. Combining rheological laws (equations (6) and (7)) thus define a piecewise continuous yield stress envelope (YSE) (Figure 3) defined as contours $\sigma' = \sigma'(x, y, t, \dot{e})$ such that

$$\sigma'(x, y, t, \dot{e}) = \text{sign}(\dot{e}) \min \{ |\sigma'[x, y, t, \dot{e}, \text{sign}(\dot{e})]|, |\sigma'(x, y, t, \dot{e})| \}$$

where $\sigma'[x, y, t, \dot{e}, \text{sign}(\dot{e})]$, $\sigma'(x, y, t, \dot{e})$ are the "brittle" and "ductile" yield stresses from (6) and (7). Owing to asymmetry of the Byerlee's law (6), the yield stress depends on the mode of deformation: $\text{sign}(\dot{e}) = 1$ for horizontal compression and $-1$ for horizontal extension.

The differential stress $\sigma(x, y)$ at a point is taken to be equal to the minimum of $\sigma'$ and $\sigma''$ computed as a function of the local strain $\dot{e}(x, y, t, \dot{e})$:

$$\sigma(x, y) = \text{sign}(\dot{e}) \min \{ |\sigma'[x, y, t, \dot{e}, \text{sign}(\dot{e})]|, |\sigma''(x, y, t, \dot{e})| \}$$

where $\sigma''(x, y, t, \dot{e})$ is the elastic differential stress according to (5). If $\sigma''$ exceeds $\sigma'$, the material is considered as ductile or brittle, depending on which rheology limits the yield strength. Equation (10) implies that the lithosphere remains elastic if imposed stress does not exceed the yield stress.

This formulation accounts for the lithosphere getting weaker when submitted to increasing horizontal forces (Figure 3a) or flexural stresses (Figure 3b) and when the crust gets thicker. Most of the upper crust remains elastic (at depths between 5 and 15–20 km). The crust is mostly ductile below 15–20 km.
Figure 4. Initial geometry and boundary conditions. Initial range is a Gaussian mount of ~400 km width and 3 km height placed on a Paleozoic lithosphere with initially 35-km-thick crust. The total width of the region is 2000 km. The 2000-km-long section is submitted to horizontal shortening \( u_0 \) corresponding to the average strain rate \( \dot{\varepsilon}_{xx} = u_0/L \). Hydrostatic (Winkler) forces are assumed at the bottom of the lithosphere.

Response to variation of the thickness of the viscous channel in the lower crust, erosion, and sedimentation at the surface) are treated as instantaneous deflections of flexible layers (Appendix A). Flow in the ductile crust is driven by the deflection of the competent layers that bound the low-viscosity channel. The normal load which is the weight of the topography \( p(x) \) and of the upper crustal layer (thickness \( h_{c1} \) and density \( \rho_{c1} \)) is applied to the surface of the lower crustal layer through the flexible competent upper crustal layer. Deformation in the lower crust is modeled as flow of a non-Newtonian fluid in a channel with variable thickness. No horizontal flow at the axis of symmetry of the range (\( x = 0 \)) is allowed. Away from the mountain range, where the channel has a nearly constant thickness, the flow is computed according to the thin channel approximation (Appendix B). Since the conditions for this approximation are not completely satisfied in the thickened region, we use a semianalytical solution for the ascending flow fed by remote channel source (Appendix C). The distance \( a \) at which the channel flow approximation is replaced by a formulation for the ascending flow, is equal to one to two thicknesses of the channel, depending on the integrated strength of the upper crust (Appendices B and C). We impose continuity of the integrated flux at the boundary between the two models.

Since the brittle-elastic-ductile (BED) rheology implies decoupling between the mantle and crust, in particular where the crust is thick, deformation of the crust is expected to be relatively insensitive to what happens in the mantle. Shortening of the mantle lithosphere is therefore neglected.

The uppermost crust deforms brittlely (Figure 3). The flow could be approximated by a viscous flow assuming some depth-averaged viscosity defined as \( \bar{\mu}_{eff} = \bar{\mu}/\bar{d} \) [Beekman, 1994]. We could also extend the solution of the equations for the horizontal flow to the stronger upper portions of the upper crust. However, owing to both higher viscosity and much lower thickness of the strong upper crustal layers, we simply assume that \( v = v(y_{13}) \) and \( u = u(y_{13}) \) for \( y \leq y_{13} \) (\( y \) is positive downward).

The various equations that define the mechanical structure of the lithosphere (equations (5)-(7) and (9)-(11)), flexure of the competent layers (equation (A1)), ductile flow in the ductile crust (equations (B5), (B6), and (C3)), erosion (equation (3) or (4)), and sedimentation at the surface (equation (12e)) are solved according to the following scheme:

\[
\begin{align*}
T, \dot{\varepsilon}, A, H, n, T_{c(k-1)} & \rightarrow ((5)-(7), (9)-(10)) \\
& \rightarrow \sigma', h_{c1}, h_{c2}, h_m \\
& \rightarrow (A1) \rightarrow w_k, T_{c(k)}, \sigma(e), y_{nm} \\
& \rightarrow ((B.5), (B.6), (C.3)) \rightarrow u_k, v_k, h_k, T_{c(k)} + BC_k \\
& \rightarrow ((3)-(4)) \rightarrow h_{k+1} \\
\end{align*}
\]

where left-hand sides are input and right-hand sides are output and BC and IC refer to boundary and initial conditions, respectively. Notation \((k)\) implies that related value is used on \(k\)th numerical step. Notation \((k - 1)\) implies that the value is taken as a predictor from the previous time step. Notation \((k + 1)\) means that the corrected value will be used in the next step. All variables are defined in the notation section.
The following continuity conditions are satisfied at the interfaces between the competent layers and the ductile crustal channel:

**Continuity of vertical velocity**

\[ v_{c1} = v_{c2} = v_m \]  

(13a)

**Continuity of normal stress**

\[ \sigma_{y_{c1}} = \sigma_{y_{c2}} = \sigma_{y_m} \]  

(13b)

**Continuity of horizontal velocity**

\[ u_{c1} = u_{c2} = u_m \]  

(13c)

**Continuity of the tangential stress**

\[ \sigma_{xy_{c1}} = \sigma_{xy_{c2}} = \sigma_{xy_m} \]  

(13d)

**Kinematic condition**

\[ \frac{\partial h}{\partial t} = v_{c2} \quad \frac{\partial w}{\partial t} = v_{c2} \]  

(13e)

Superscript plus and minus refer to the values on the upper and lower interfaces of the corresponding layers, respectively. The subscripts \( c1, c2 \), and \( m \) refer to the strong crust ("upper"), ductile crust ("lower"), and mantle lithosphere, respectively. Power law rheology results in self-lubrication and concentration of the flow in the narrow zones of highest temperature, that is, near the Moho. Therefore there is little difference between assuming no-slip or free slip for the ductile crust.

The spatial steps used in calculations are \( dx = 2 \text{ km}, dy = 0.5 \text{ km} \). The requirement of stability of integration of the diffusion equations (3), (4) \( (dt < 0.5dx^2/k) \) implies a maximum time step of 2000 years for \( k = 10^3 \text{ m}^2/\text{yr} \) and of 20 years for \( k = 10^4 \text{ s} \text{ m}^2/\text{yr} \). It is less than the relaxation time for the lowest viscosity value \( (-50 \text{ years for } \mu = 10^{19} \text{ Pa s}) \). We chose a time step of 20 years in all computations.

**Experiments and Results**

**Description of Experiments**

We conducted series of experiments in which a 2-D section of a continental lithosphere, loaded with some initial range, is submitted to horizontal shortening (Figure 4). The shortening rate in the crust is assumed equal to that in the mantle \( (u_c = u_m) \). Our goal was to validate the coupled regime described in the first section and to check whether it can allow mountain growth. Our purpose was not to explain how some small initial irregularity of the topography might develop into a mountain belt. This question is beyond the scope of this study because fault initiation and propagation were not considered. We only address the problem of the growth and maintenance of a mountain range once it has reached some mature geometry.

We thus consider a 2000-km-long portion of lithosphere loaded by an initial major topographic irregularity. We chose a 300- to 400-km-wide "Gaussian" mountain (a Gaussian curve with variance \( \sigma = 100 \text{ km} \)). The range has a maximum elevation of 3000 m and is initially regionally compensated. The initial geometry of the Moho was computed according to the flexural response of the competent cores of the crust and upper mantle and neglecting viscous flow in the lower crust [Burov et al., 1990]. In this computation the possibility of internal deformation of the mountain range or of its crustal root is neglected. The section is then submitted to horizontal shortening at rates from about 1 mm/yr to a few centimeters per year. These rates largely span the range of most natural large-scale examples of active intracontinental mountain range. Each experiment is conducted over about 15–20 m.y. with internal time step of 20 years. The geometries of the different interfaces (topography, upper crust–lower crust, Moho, basement-sediment in the foreland) are computed for each time step. We also determine the rate of uplift of the topography, \( du/dt \); the rate of tectonic uplift or subsidence, \( du/dt \); the rate of denudation or sedimentation, \( de/dt \) (Figure 5), as well as stress, strain, and velocity fields in the lithosphere. The relief of the range, \( \Delta h \), is defined as the difference between the elevation at the crest \( h(0) \) and in the lowlands at 500 km from the range axis, \( h(500) \).

In the case where there are no initial topographic or rheological irregularities, the medium has homogeneous properties and therefore thickens homogeneously. There are no horizontal or vertical gradients of strain so that no mountain can form. If the medium is initially loaded with a mountain range, the flexural stresses \( (300–700 \text{ MPa}, \text{ Figure 6a}) \) can be 3–7 times higher than the excess pressure associated with the weight of the range itself \( (-100 \text{ MPa}) \). The deviatoric stresses may be higher than the lithostatic pressure up to depths of 15–20 km and remain comparable with the pressure up to depths of order 100 km (depending on the rheology). As a result the competent cores are thinnest in the area below the range (Figure 6b).

Horizontal shortening of the lithosphere tends therefore to be absorbed preferentially by strain localized in the weak zone beneath the range. In all experiments the system evolves rapidly during the first 1–2 m.y. because the initial geometry is out of dynamic equilibrium. After this reorganization some kind of dynamic equilibrium settles in which the viscous forces due to flow in the lower crust also participate to the support of the load.

**Case 1, No Surface Processes: "Subsurface Collapse"**

In the absence of surface processes the lower crust is extruded from under the high topography (Figure 7). The crustal root and the topography spread out laterally (Figure 8a). Horizontal shortening leads to a general thickening of the medium, but the tectonic uplift below the range is smaller than below the lowlands so that the relief of the range, \( \Delta h \), decays with time (Figure 8b). The system thus evolves toward a regime of
Figure 6a. Differential stresses in the lithosphere for a particular experiment \((k = 7500 \text{ m}^2/\text{yr}, \dot{\varepsilon}_{xx} = 0.7 \times 10^{-15} \text{ s}^{-1}, \dot{u} = 4.4 \text{ cm/yr}, t = 5 \text{ m.y.})\). Note flexural stresses and the concentration of stress in the lower crust beneath the range. Pluses and minuses refer to domains in compression and tension, respectively.
Figure 6b. Strength variations in the lithosphere for the same experiment as in Figure 6a. In order to distinguish between the competent and weak domains, the stresses are compared to the elastic stress $\Delta \sigma_e$ that would be expected for the same strain. White areas ($\Delta \sigma / \Delta \sigma_e > 0.9$) correspond to elastic “cores.” The dark areas represent weak domains where the YSE has been reached (in the brittle or ductile regime). Note that the elastic cores are significantly reduced beneath the range, where local isostasy can be approached. The effect of weakening of the lithosphere by surface load favors localization of deformation in the vicinity of the range.
homogeneous deformation with a uniformly thick crust. In the particular case of a 400-km-wide and 3-km-high range it takes about 15 m.y. for the topography to be reduced by a factor of 2. If the medium is submitted to horizontal shortening, the decay of the topography is even more rapid due to strain weakening. These experiments actually show that assuming a standard rheology for the crust, no intrinsic strain softening properties of the material, and no particular mantle dynamics, a range should collapse in the long term, as a result of subsurface deformation, even with horizontal shortening of the lithosphere. This regime in which a range decays by lateral extrusion of the lower crustal root might be called "subsurface collapse."

Case 2, No Shortening: "Erosional Collapse"
If erosion is relatively active (k of the order of $10^4$ m$^2$/yr), the topography of the range is washed out rapidly (Figure 9a). Isostatic readjustment compensates for only a fraction of the denudation, and the elevation in the lowland increases as a result of crustal thickening essentially due to sedimentation (Figure 9b). Although mechanical collapse of the crustal root also participates in the process of decay of the range, we may call this regime "erosional collapse." The time constant associated with the decay of the relief in this regime depends on the mass diffusivity (Figure 9c). For $k = 10^4$ m$^2$/yr, denudation rates are of the order of 1 mm/yr at the beginning of the experiment (Figure 9b), and the initial topography is halved after about 5 m.y. For $k = 10^3$ m$^2$/yr the range topography is halved after about 15 m.y. Once the crust and Moho topographies have been smoothed by surface processes and subsurface deformation, the system evolves again toward the regime of homogeneous thickening.

Case 3, Shortening and Erosion: "Mountain Growth"
Mountain growth can occur provided that denudations are not too rapid nor too slow compared to the shortening rate. We performed a set of experiments starting from points in the "subsurface collapse" regime, and we then gradually increased the intensity of erosion, keeping the shortening rate constant. If erosion is not active enough, the range decays as a result of subsurface deformation. At some critical value of k a regime of

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**Figure 7.** Outward flow pattern in the lower crust in the case of no erosion. The crustal root below the range is extruded laterally driven by the shear stresses due to the slopes of the topography and of the Moho. Velocity $u_x$ at $y = y_{13}$ associated with overall compression is subtracted. In this experiment, the 2000-km-long section is submitted to 6.3 mm/yr horizontal shortening.

**Figure 8a.** A case example of "subsurface collapse" when no erosion is applied. Elevation at crest of the range decays as a result of the subsurface deformation as shown in Figure 7. The 2000-km-long section is submitted to 6.3 mm/yr horizontal shortening. Evolution (top) of topography and (bottom) of Moho from 0 to 10 m.y. The different curves are labeled according to time (0, 1, 2, 3, 4, 5, 10 m.y.).

**Figure 8b.** Evolution of elevation $h$ at axis of the range ($x = 0$ km) and in the lowland ($x = 500$ km) as a function of time. Flexural and flow adjustment occurs during the first 1–2 m.y.
dynamical coupling can settle in which the relief of the range can grow (Figure 10a). Similarly, we may start from a state of “erosional collapse,” keep the rate of erosion constant, and gradually increase the rate of shortening. We get that for low shortening rates, erosion can still erase the range topography, but at some critical value of the shortening rate a coupled regime can settle (Figure 10a). In the coupled regime the lower crust flows toward the crustal root (Figure 10b), and the resulting flux exceeds the amount of material removed from the range by surface processes. Tectonic uplift below the range exceeds denudation (Figure 10c) so that the elevation of the crest increases with time (Figure 10d). In this “mountain growth” regime the distribution of deformation remains heterogeneous in the long term. High strains in the lower and upper crust are localized below the range allowing for crustal thickening. The crust in the lowland also thickens owing to sedimentation but at a smaller rate than beneath the range (Figure 10c). Figure 11 shows that the rate of growth of the elevation at the crest, \( dh/dt(x = 0) \), for different experiments in the “mountain growth regime.” First we see that for a given experiment the evolution is far from monotonic in general. There are oscillations probably related to fluctuations around a dynamic equilibrium in which eroded material and inward flow in the lower crust rate by a constant ratio. Such a dynamic equilibrium occurs for experiment 1. If denudation is too rapid, the topography flattens. Since the slopes of the topography and Moho are reduced inward flow in the lower crust can proceed more rapidly, inducing a renewed uplift. Such oscillations are particularly clear in experiment 3. Comparison of the different experiments indicates that the behavior of the system is very sensitive to small changes of parameters. We did not explore the dynamical behavior of the system in the coupled regime, but we suspect that some chaotic behavior may arise.

The results for different experiments are summarized in Figure 12. As a convention a given experiment, corresponding to a given mass diffusivity and a given shortening rate (or...
Localized stable growth of the range (compression) 

\[ u = 44 \text{ mm/yr} \left( E_{\text{eq}}=0.76-15 \text{ s}^{-1} \right) + \text{linear erosion (}K_s=7500 \text{ m}^2/\text{yr}\right) \]

Figure 10a. A case example of "mountain growth." Coupling between erosional and tectonic processes results in a slow localized growth of a range. In this example, the section is submitted to 44 mm/yr of shortening, and erosion is relatively intense with a mass diffusivity of \( k = 7500 \text{ m}^2/\text{yr} \). Evolution (top) of topography and (bottom) of crustal root.

average strain rate), is defined to be in the "mountain growth" regime if the relief of the range increases at 5 m.y., which means that elevation at the crest (\( x = 0 \)) increases more rapidly than the elevation in the lowland (\( x = 500 \text{ km} \)):

Figure 10b. Flow pattern in lower crust. Dashed line in lower crust corresponds to deflection of lower crust. Solid line is the bottom of competent upper crust. Bending stresses result in thermomechanical "erosion" of bottom of mechanical upper crust. A constant horizontal velocity, corresponding the horizontal velocity at the bottom of the competent crust (\( y_{13} \)) is subtracted.

Figure 10c. Rates of denudation (\( de/dt \)), surface uplift (\( dh/dt \)), and tectonic uplift (\( du/dt \)). Notations \( dh_3 \), \( dh_5 \), \( dh_{10} \), etc., correspond to \( t = 3, 5, \) and 10 m.y., etc., respectively. Thick grey line shows initial topography.

Figure 10d. Evolution of elevation \( h \) at the axis of range (\( x = 0 \text{ km} \)) and in the lowland (\( x = 500 \text{ km} \)) as a function of time.
Because of the complex dynamic shown in Figure 11, very close experiments appear to be labeled differently. Nevertheless, the “mountain growth,” “subsurface collapse,” and “erosional collapse” domains can be approximately drawn on this plot. Higher strain rates lead to a decrease of the effective viscosity ($\mu_{eff}$, proportional to $e^{1/\alpha - 1}$) of the non-Newtonian lower crust so that a more rapid erosion is needed to allow the feedback effect due to surface processes. The mountain growth domain thus corresponds to higher mass diffusivities for higher strain rates (Figure 12).

**Coupled Regime and Graded Geometries**

In the coupled regime the topography of the range can be seen to develop into a nearly parabolic graded geometry (Figure 10a). This graded form is attained after 2-3 m.y. and reflects some dynamic equilibrium with the topographic rate of uplift being nearly constant over the range (Figure 10c). Rates of denudation and of tectonic uplift can be seen to be also relatively constant over the range domain (Figure 10c). Geometries for which the denudation rate is constant over the range are nearly parabolic since they are defined by

$$de/dt = k \frac{d^2h}{dx^2} = C_2,$$

with $C_2$ being some constant. Integration of this expression yields a parabolic expression for $h$:

$$h = \frac{C_2}{2k} x^2 + C_1 x + C_0.$$

The graded geometries obtained in the experiment slightly deviate from parabolic curves because they do not exactly correspond to uniform denudation over the range. This simple consideration does, however, suggest that the overall shape of graded geometries is primarily controlled by the erosion law.

We made computations with nonlinear erosion laws (equation (4)). For a given shortening rate, experiments that correspond to similar erosion rates over the range lead to the same evolution (“erosional collapse,” “subsurface collapse,” or “mountain growth”) whatever the erosion law. It thus appears that the emergence of the coupled regime does not depend on a particular erosion law but rather on the intensity of erosion relative to the effective viscosity of the lower crust. By contrast the graded geometries obtained in the mountain growth re-
Figure 13. Comparison of different geometries obtained in the coupled regime for various models of erosion. The section is submitted to 44 mm/yr horizontal shortening. Thick grey line shows initial topography. After rapid evolution during the first 2 m.y. the range topography gradually increases with time. (a) Second-order nonlinear erosion model with asymmetric boundary conditions (vertical force $Q_c = 2 \times 10^{12}$ N per unit length is applied on the left edge of plate). The curves correspond to 1, 2, 3, 4, 5, and 10 m.y. (b) First-order nonlinear erosion model. The curves correspond to 1, 2, 3, 4, and 5 m.y. (c) Linear diffusion model of erosion. The curves correspond to 1, 2, 3, 4, and 5 m.y.

The experiments shown above have been conducted assuming a quartz rheology for the whole crust that is particularly favorable for channel flow in the lower crust. In order to show that the coupled regime of mountain growth can also occur assuming a less ductile lower crust, we also conducted some experiments assuming more basic lower crustal compositions (diabase, quartz-diorite) (Table 1). Even with a relatively strong lower crust the coupled regime allowing for mountain growth can settle (Figure 14). The effect of a less viscous lower crust is that for a given shortening rate, lower rates of erosion are required to allow for the growth of the initial mountain. The domain defining the "mountain growth" regime in Figure 12 would thus be shifted toward smaller mass diffusivities when a stronger lower crust is considered. The graded shape obtained in this regime does not differ from that obtained with a quartz rheology for the lower crust (Figure 14). Estimates of the yield strength of the lower crust near the Moho boundary for thermal ages from 0 to 2000 m.y. made by Burov and Diament [1995] suggest that whatever rheology is considered, a crust thicker than about 40-50 km implies a low-viscosity channel in the lower crust. The regime that we have evidenced in our numerical simulations could therefore exist for a wide range of conditions.

Comparison With the Tien Shan

Although the Tien Shan may not appear as a generic mountain range, we compare our modeling with that particular range because rates of deformation and of erosion have been estimated from previous studies [e.g., Avouac et al., 1993; F. Métivier and Y. Gaudemer, Mass transfer between eastern Tien Shan and adjacent basins: Constraints on regional tectonics and topography, submitted to Geophysical Journal International, 1996] and because the range has a relatively simple 2-D geometry. The Tien Shan is the largest and most active mountain range in central Asia (Figure 1a). It extends for nearly 2500 km between the Kyzil Kum and Gobi deserts, with peak rising to more than 7000 m. The high level of seismicity in this century [Molnar and Deng, 1984] and deformation of Holocene alluvial formations [Avouac et al., 1993] would indicate a rate of shortening of the order of 1 cm/yr. In fact, the shortening...
Clockwise rotation of the Tarim Basin with respect to Dzungaria and Kazakhstan would be responsible for this westward increase of shortening rate as well as of the increase of the width of the range (Figure 1a) [Chen et al., 1991; Avouac et al., 1993]. The gravity studies by Burov [1990] and Burov et al. [1990] also suggest westward decrease of the integrated strength of the lithosphere. The westward increase of the topographic load and strain rate could be responsible for this mechanical weakening. The geological record suggests a rather smooth morphology with no great elevation differences and low elevations in the Tertiary and that the range was reactivated in the middle Tertiary, probably as a result of the India-Asia collision [e.g., Tapponnier and Mohar, 1979; Hendrix et al., 1992]. Fission track ages from detrital apatite from the northern and southern Tien Shan would place the reactivation at about 20 m.y. [Hendrix et al., 1994; E. Sobel and T. A. Dumitru, Exhumation of the margins of the western Tarim Basin during the Himalayan orogen, submitted to Tectonics, 1995]. Such an age is consistent with the middle Miocene influx of elastic material and more rapid subsidence in the forelands [Hendrix et al., 1992]. The present difference of elevation of about 3000 m between the range and the lowlands (Figure 1b) would therefore indicate a mean rate of uplift of the topography, during the Cenozoic orogeny, of the order of 0.1–0.2 mm/yr. The foreland basins have collected most of the material removed by erosion in the mountain. Sedimentary isopachs indicate that 1.5 ± 0.5 × 10^6 km^3 of material would have been eroded during the Cenozoic orogeny (F. Métrivier and Y. Gaudemer, submitted manuscript, 1996), implying erosion rates of 0.2–0.5 mm/yr on average. The tectonic uplift would thus have been of 0.3–0.7 mm/yr on average. On the assumption that the range is approximately in local isostatic equilibrium [Burov et al., 1990; Ma, 1987], crustal thickening below the range has absorbed 1.2–4 × 10^6 km^3 (Métivier and Gaudemer, submitted manuscript, 1996). Crustal thickening would thus have accommodated 50–75% of the crustal shortening during the Cenozoic orogeny, with the remaining 25–50% having been fed back to the lowlands by surface processes. Given the geometry of the range, the 0.2–0.5 mm/yr denudation rate implies a mass diffusivity of the order of 2 × 10^3 to 10^4 m^2/yr. These values actually place the Tien Shan in the "mountain growth" regime on the plot in Figure 12. We therefore conclude that the localized growth of a range like the Tien Shan could actually result from the coupling between surface processes and horizontal strains. We do not dispute the possibility for a complex mantle dynamics beneath the Tien Shan as has been inferred by various geophysical investigations [Vinnik and Saibpekova, 1984; Makeyeva et al., 1992; Roecker et al., 1993], but we contend that this mantle dynamics has not necessarily been the major driving mechanism of the Cenozoic Tien Shan orogeny.

**Conclusions and Implications**

This study demonstrates that denudation could exert some control on the development of an intracratonic mountain belt. Depending on the intensity of the surface processes, horizontal compression of a 2-D section of continental lithosphere with some initial topographic irregularity can lead either to strain localization below a growing range or to distributed thickening. Homogeneous thickening occurs when erosion is either too strong, in which case any topographic irregularity can be rapidly washed out by surface processes, or too weak, in which case the crustal root below the range spreads out laterally with formation of a flat "pancake-shaped" topography. In our modeling, mountain growth results from dynamical coupling between the advection at the surface by surface processes and at

![Figure 14](image-url)
Deformation of a continent may thus be very sensitive to surface processes. The way central Asia has absorbed indentation of India may somehow reflect this sensitivity. Numerical models of continental indentation based on continuum mechanics, but that neglect surface processes, predict a broad zone of crustal thickening, resulting from nearly homogeneous strain that would have propagated away from the indenter [England and Houseman, 1986; Houseman and England, 1986; Vilotie et al., 1982]. In fact, long-lived zones of localized crustal shortening have been maintained (in particular, along the Himalaya, at the front of the indenter, and the Tien Shan, well north of the indenter), and broad zones of thickened crust have resulted from sedimentation rather than from horizontal shortening (in particular in the Tarim Basin and to some extent in some Tibetan basins such as the Tsaidam). Present kinematics of active deformation in central Asia corroborates a highly heterogeneous distribution of strain. The \( 5 \text{ cm/yr} \) convergence between India and stable Eurasia is absorbed by lateral extrusion of Tibet and crustal thickening with crustal thickening accounting for about \( 3 \text{ cm/yr} \) [Avouac and Tapponnier, 1993]. About \( 2 \text{ cm/yr} \) would be absorbed in the Himalayas and \( 1 \text{ cm/yr} \) in the Tien Shan (Figure 15). The indentation of India into Eurasia has thus induced localized strain below two relatively narrow zones of active orogenic processes, while minor deformation has been distributed elsewhere. Surface processes might be partly responsible for this highly heterogeneous distribution of deformation. First, active thrusting along the Himalaya and in the Tien Shan may have been sustained during most of the Cenozoic thanks to continuous erosion. Second, the broad zone of thickened crust in central Asia has resulted in part from the redistribution of the sediments eroded from the localized growing reliefs. Moreover, it should be observed that the Tien Shan experiences a relatively arid intracontinental climate, while the Himalaya is exposed to a very erosive monsoonal climate. Although rheological differences might also be advocated, this climatic disparity may explain why the Himalaya absorbs twice as much horizontal shortening as the Tien Shan, while tectonic forces are probably similar. In addition, the nearly equivalent climatic conditions on the northern and southern flanks of the Tien Shan might have favored the development of a nearly symmetrical range. By contrast the much more erosive climatic conditions on the southern than on the northern flank of the Himalaya may have favored the development of systematically south vergent structures. While the Indian upper crust would have been delaminated and brought to the surface of erosion by north dipping thrust faults, the Indian lower crust would have flowed below Tibet (Figure 15). Surface processes might therefore have facilitated injection of indiyan lower crust below Tibet as was proposed by Zhao and Morgan [1985]. This would explain crustal thickening of Tibet with minor horizontal shortening in the upper crust and minor sedimentation.

We thus suspect that climatic zonation in Asia has exerted some control on the spatial distribution of the intracontinental strain induced by the India-Asia collision. The interpretation of intracontinental deformation should not be thought of only in terms of boundary conditions induced by global plate kinematics but also in terms of global climate. Climate might therefore be considered as a forcing factor of continental tectonics.

Appendix A: Model of Flexural Deformation of the Competent Cores of the Brittle-Elasto-Ductile Crust and Upper Mantle

Deformation of the competent layers in the crust and mantle, as defined from the BED rheology (Figures 3 and 4) in response to redistribution of loads by surface processes or flow in the lower crust, is computed using plate equilibrium equations [Burov and Diament, 1995]:

\[
\begin{align*}
\frac{\partial}{\partial x} \left[ \frac{\partial}{\partial x} \left( \frac{E}{12(1-\nu^2)} \left( \frac{\partial^2 w(x, t)}{\partial x^2} \right)^2 + \frac{\partial w(x, t)}{\partial x} \right) \right] + p_\nu (w(x, t) - p_\nu (x, t) = 0 \quad (A1a)
\end{align*}
\]

\[
\begin{align*}
\tilde{T}_1 (\phi) &= \left[ \tilde{M}_1 (\phi) \left( \frac{\partial^2 w(x, t)}{\partial x^2} \right)^{-1} \right]^{1/3} \quad (A1b)
\end{align*}
\]

\[
\begin{align*}
\tilde{M}_1 (\phi) &= - \sum_{i=1}^{n} \sum_{j=1}^{m} \int_{\gamma_i (\phi)} \gamma_j (\phi) \sigma_{ij} \phi) dy \quad (A1c)
\end{align*}
\]

\[
\begin{align*}
\tilde{T}_1 (\phi) &= \sum_{i=1}^{n} \sum_{j=1}^{m} \int_{\gamma_i (\phi)} \sigma_{ij} \phi) dy \quad (A1d)
\end{align*}
\]

\[
\begin{align*}
\sigma_{ij} (\phi) &= \text{sign} (\sigma_{ij}) \min \left[ |\sigma_{ij}|, \sigma_{ij}^{\phi} (\phi) \right] \quad (A1e)
\end{align*}
\]
where \( w = w(x, t) \) is the vertical plate deflection (related to the regional isotropic contribution to tectonic uplift \( du_n \)), as \( du_n = w(x, t) - w(x, t - dt) \); \( \mathbf{f} = \{ x, y, w, w', w'', t \} \); \( y \) is positive downward; \( y'_n(x) = y - y_n(x) \); \( y_n(x) \) is the depth to the \( n \)th neutral (i.e., stress-free, \( \sigma_{xx}(y_n) = 0 \)) plane; and \( y'_1(x) = y_1(x), y'_n(x) = y_n(x) \) are the respective depths to the lower and upper low-strength interfaces (see Figure 3b). The strength \( \sigma \) is defined by equation (9); \( n = 1 \) refers to the competent crust and \( n = 2 \) refers to the competent mantle; \( m \) is the number of "welded" (continuous \( \sigma_{xx} \)) sublayers in the \( n \)th detached layer; \( p_w \) is a restoring stress \( (p_w = (\rho_g - \rho_i)g) \) and \( p^+ \) is a sum of surface and subsurface loads. The most important contribution to \( p^+ \) is from the load of topography, that is, \( p^+ = pgh(x, t) \), where the topographic height is \( h(x, t) \). The thickness of the \( n \)th competent layer is \( y_n' - y'_n(x) = \Delta h_n(x) \). The term \( \partial^2 w / \partial x^2 \) is directly proportional to the radius of plate curvature \( R_{xy} \). Thus the higher the local curvature of the plate, the lower the integrated strength of the lithosphere. The integrals in (A1) are defined through the constitutive laws (5)-(7) and (9)-(10) relating the stress \( \sigma_{xx} \) and strain \( e_{xx} = \frac{\sigma_{xx}}{\mu} \) in a given segment \( \{ x, y \} \) of a plate. Note that the value of the effective elastic thickness \( T_e(x, t) \) holds only for the given solution for plate deflection \( w \). The nonlinear equations (A1a)-(A1f) are computed using an iterative approach based on finite difference approximation (block matrix presentation) with linearization by Newton's method [Burov and Diament, 1992]. The procedure starts from calculation of elastic prediction \( w_e(x) \) for \( w(x) \), that gives predicted \( w_e(x) \), \( (\partial w / \partial x)(x) \), \( (\partial^2 w / \partial x^2) \), \( u \), \( v \), or \( w \), \( u \), \( v \) used to derive solutions for \( y_1'(x), y_n'(x) \), and \( y_n'(x) \) that satisfy (5), (6), (7), and (10). Corrected solutions for \( M_x \) and \( T_e \) are deduced and used to obtain \( T_e \) for the next iteration. At this stage we use gradual loading technique to avoid numerical oscillations. The accuracy is checked through back-substitution of the current solution in (A1) and calculation of the discrepancy between the right and left sides of (A1). For the boundary conditions on the ends of the plate we use commonly inferred combination of plate boundary shearing force \( Q_0(0) \),

\[
\sigma^{ij}_{xx}(\phi) = \left( \frac{y_i(x) \partial w(x, t)}{\partial x^2} \right) \right)_E(1 - \nu^2)^{-1} \quad (A1f)
\]

and plate boundary moment \( M_0(0) \) (in the case of broken plate) and \( w = 0, \partial w / \partial x = 0 \) (and \( h = 0, \partial h / \partial x = 0 \)) at \( x \to \pm \infty \).

Appendix B: Model of Flow in the Ductile Crust

Flow in the low-viscosity crustal channel is computed in a way similar to the Couette flow model of Lobkowsky [1988], Lobkowsky and Kerchman [1991] (hereafter referred as LK), or Bird [1991]. Our formulation can allow computation of different types of flow (symmetrical, Poiseuille, Couette). In the numerical experiments shown in this paper we consider only cases with a mixed Couette/Poiseuille/symmetrical flow, but we also checked our formulation by comparison with that of LK for a Couette flow. The low-viscosity crustal channel is bounded by the competent layers in the crust at the top and in the mantle at the bottom. Flexure of these layers will drive flow in the lower crust. In turn, changes of the thickness of the lower crustal channel, \( \Delta h_{c2} = \Delta h_0(x, 0) + \bar{h} + w \), will be compensated by deflection of the Moho, which depth is \( h_c(x, t) = T_c(x, t) = \Delta h_{c2} + h_0(x, 0) \) and of the upper crust. These deflections are assumed instantaneous and are computed according to Appendix A. Note that the topography of the upper boundary of the lower crustal channel, \( \bar{h} \), does not mimic topographic variation at the surface, \( h(x, t) = h(x, t - dt) = \Delta u = dy_{13} \), where \( dy_{13} = y_{13}(x, t) - y_{13}(x, t - dt) \) is the change of the depth of the lower boundary of the elastic core in the upper crust due to changes of deviatoric stresses (as shown in Figure 3b).

The equations governing the creeping flow of an incompressible fluid, in Cartesian coordinates, are, in addition to equations (7) and (8),

\[
- \partial \sigma_{xx} / \partial x + \partial \tau_{xy} / \partial y + F_x = 0; \quad - \partial \sigma_{yy} / \partial y + \partial \tau_{xy} / \partial x + F_y = 0
\]

\[
\sigma_{xx} = -\tau_{xx} + p = -2\mu \frac{\partial u}{\partial x} + p \quad (B1a)
\]

\[
\sigma_{yy} = \tau_{yy} = \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)
\]

\[
\tau_{xy} = -\tau_{yx} = \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (B1b)
\]

Where \( \mu \) is the depth-dependent effective viscosity, \( p \) is pressure, \( u \) and \( v \) are the horizontal and vertical components of the velocity \( \mathbf{v} \), respectively; \( F \) is the body force due to gravity; \( u = \alpha \partial \mathbf{v} / \partial y \) is the horizontal component of velocity of the differential movement in the ductile crust; \( v = -\partial \mathbf{v} / \partial x \) is its vertical component; and \( \partial u / \partial y = \varepsilon_{xy} \) is a component of shear strain rate due to the differential movement of the material in the ductile crust (the components of the strain rate tensor are, consequently, \( \varepsilon_{11} = 2\partial u / \partial x \); \( \varepsilon_{12} = \partial u / \partial y + \partial v / \partial x \); \( \varepsilon_{22} = 2\partial v / \partial y \)).

Within the low-viscosity boundary layer of the lower crust, the dominant basic process is simple shear on horizontal planes, so the principal stress axes are dipped approximately \( \pi / 2 \) from \( x \) and \( y \) (hence \( \sigma_x \) and \( \sigma_y \) are approximately equal). Then, the horizontal component of quasi-static stress equilibrium equation \( \partial \sigma_{xx} + \rho g = 0 \), where tensor \( \sigma \) is \( \sigma = \sigma - P I \) (I is identity matrix), can be locally simplified yielding thin layer approximation [e.g., Lobkowsky, 1988; Bird, 1991]:

\[
\frac{\partial \sigma_{xx}}{\partial y} = \frac{\partial \sigma_{xy}}{\partial x} - F_x = - \frac{\partial \tau_{xy}}{\partial x} \quad (B2)
\]

An effective shear strain rate can be derived \( \varepsilon_{xy} = \sigma_{xy} / 2\mu_0 \); therefore, according to constitutive relation (7), horizontal velocity \( u \) in the lower crust is

\[
u \bar{y} = \int_0^x 2\mu_0 \frac{\partial \sigma_{xx}}{\partial x} \bar{y} + C_1 = \int_0^x 2\mu_0 A^* \exp \left( -H^* / RT(y) \right) \left| \tau_{xy} \right|^{-1} \sigma_{xy} \bar{y} + C_1 \quad (B3)
\]
Here $\bar{y} = y - y_{13}; y_{13} = y_{13}(\phi)$ is the upper surface of the channel defined from solution of the system (A1).

$C_4$ is an arbitrary constant of integration defined from the velocity boundary conditions (Figure 4); $\tau_{xy}$ is defined from lithosphere (Appendix A) are in accordance with the condition for ductile flow: $x \to \pm \infty$: $u_{c2} = u_c; u_{c2} = u_m$; $\partial p/\partial x = 0; \partial p/\partial y = \bar{p}_c; p = \bar{p}_c$.

The major perturbation to the stress field is caused by slopes of crustal interfaces $\alpha - 9h/\partial x$ and $\beta - \partial w/\partial x$. These slopes are controlled by flexure, isostatic adjustments, surface erosion, and modification of the interfaces by stress and temperature (thermomechanical weakening). In the assumption of small plate deflections the horizontal force associated with variation of the gravitational potential energy due to deflection of Moho $(w)$ is $\varphi_c g \tan(\beta) - \varphi_c g \sin(\beta) - \varphi_c g \partial w/\partial x$; the vertical component of force is $-\varphi_c g \cos(\beta) - \varphi_c g (1 - \partial w/\partial x) - \varphi_c g$. The horizontal and vertical force components due to slopes of the upper walls of the channel are $\varphi_c g \tan(\alpha) - \varphi_c g \partial w/\partial x$ and $\varphi_c g \cos(\beta) - \varphi_c g (1 - \partial h/\partial x)$, respectively. The equation of motion (Poiseuille/Couette flow) for a thin layer in the approximation of lubrication theory will be

$$\begin{align*}
\frac{\partial \tau_{xy}}{\partial y} - \frac{\partial \tau_{xy}}{\partial x} &= \frac{\partial p}{\partial y} - \varphi_c g \left(1 - \frac{\partial (h + w)}{\partial x}\right) \\
\frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} &= -\varphi_c g \left(1 - \frac{\partial (h + w)}{\partial x}\right) \\
\frac{\partial u_{c2}}{\partial x} + \frac{\partial v_{c2}}{\partial y} &= 0,
\end{align*}$$

where $p = \varphi_c(x) + \bar{p}_c g (\bar{y} + y_{13} + h)$ and $\varphi_c$ is averaged crustal density. In the case of local isostasy, $w$ and $\partial w/\partial x$ are approximately 4 times $(\varphi_c/\mu(\varphi_c - \varphi_{m}) \approx 4)$ greater than $h$ and $\partial h/\partial x$, respectively. The pressure gradient due to deflection of the Moho is $\varphi_m g \partial (h + w)/\partial x$. Accounting for the difference of density between the crust and mantle, the effective pressure gradient in the crust is $(\varphi_m - \bar{p}_c) g \partial (h + w)/\partial x$, with $w$ being equal to $h(\varphi_m - \bar{p}_c)/\varphi_m$. In the case of regional compensation (as computed from the system (A1)) the difference between $h$ and $w$ is reduced by a factor 2 to 3.

Equations (B3) and (B4) imply

$$\begin{align*}
\frac{\partial \tau_{xy}}{\partial y} &= \frac{\partial p}{\partial x} - \varphi_c g \left(1 - \frac{\partial (h + w)}{\partial x}\right) \\
\frac{\partial \tau_{xy}}{\partial y} &= \varphi_c g \left(1 - \frac{\partial (h + w)}{\partial x}\right),
\end{align*}$$

Assuming small deflections $(w/T_e \ll L/T_e, \bar{h} \sim 0.2 - 0.5w$, where $L$ is the length of the plate), we get $1 - \partial (h + w)/\partial x \sim 1$.

Depth integration of (B5) yields the longitudinal and vertical components of the velocity in the lower crust. For example, we have

$$u = \int_0^{h_2 - n_h} 2^A^* \exp \left(-H^*/RT(\bar{y})\right) |\tau_{xy}|^{n-1} \tau_{xy} \, \bar{y}$$

$$v = \int_0^{h_2 - n_h} \frac{\partial \bar{y}}{\partial x}$$

The later equation gives the variation of thickness of the ductile channel in time (equal to the difference between the vertical flow at the top and bottom boundaries).

The temperature, which primarily controls the effective viscosity of the crust, is much lower in the uppermost and middle portions of the upper crust (at 10–15 km depth). As a result, the effective viscosity of the middle portions of the upper crust is 2–4 orders higher than that of the lower crust $(10^{14} \text{ to } 10^{23} \text{ Pa s})$ compared to $10^{19} \text{ to } 10^{20} \text{ Pa s}$, equations (7) and (8)). Therefore we can consider the reaction of the lower crust to deformation of the upper crust as rapid. For numerical reasons, we cut the interval of variation of the effective viscosity at $10^{19}$ to $10^{20} \text{ Pa s}$. Solution for the channel flow implies that the channel is infinite in both directions. In our case the channel is semi-infinite because of the requirement $u = 0$ at $x = 0$. Velocity boundary conditions are assumed on the upper and bottom interfaces of the lower crustal channel. Free flow is the inherent lateral boundary condition.

Appendix C: Analytical Formulation for Ascending Crustal Flow

In this appendix we present computation of the flow under the range, where the thin channel approximation of Appendix B may not be valid. In a general case of noninertial flow (low Reynolds number) the flow beneath the mountain range can be computed, assuming symmetry with respect to the axis of the range, from the system [Fletcher, 1988; Hamilton et al., 1995]

$$\begin{align*}
o &= \rho_c F_s - \frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left(2\mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right) \\
o &= \rho_c F_s - \frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left(2\mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right),
\end{align*}$$

We define $\partial p/\partial x = \partial p/\partial y + g(\rho_c 2\partial w/\partial x + \rho_c \partial (du)/\partial x)$, $du = dh$, and $\partial p/\partial y = \partial p/\partial y - g\partial p$, where $\bar{p}$ is dynamic, or modified, pressure.

The flow is assumed to be Couette/Poiseille flow (Appendix B) away from the symmetry axis (at a distance $a_1$); $a_1$ is equal to 1–2 thicknesses of the channel, depending on channel thickness-to-length ratio. In practice, $a_1$ is equal to the distance at which the equivalent elastic thickness of the crust (as defined by Burov and Diament [1995]) becomes less than 5 km. When the elastic thickness of the crust is so thin, we assume that the upper boundary of the ductile lower crust can be considered free of shear stress. This assumption makes the computation easier. Another more realistic boundary condition would probably modify the flow only in a thin zone at top of the low-viscosity channel in the lower crust.

The remote feeding flux $q$ at $x \to \pm a_1$ is equal to the value of flux obtained from depth integration of the channel source (Couette flow). The flux $q$ is determined as $q = f u \, dy$ (per
unit length in z direction). This flux feeds the growth of
the topography and deepening of the crustal root. Combination
of two flow formulations is completed using continuity equations
[Huppert, 1982]
\[
\int \frac{\partial u}{\partial y} dy + \frac{\partial}{\partial x} \left( \int u \; dy \right) = 0 = \frac{\partial (h + w)}{\partial t} + \frac{\partial q}{\partial x} \tag{C2a}
\]
\[
\lim_{x \to \pm \infty} q = q \big|_{x=\pm \infty} \tag{C2b}
\]
\[
\int_{0}^{\alpha(x)} (\bar{h} + w) \; dx + \int_{\alpha(x)}^{\infty} (\bar{h} + w) \; dx = q t \tag{C2c}
\]
where \( \theta = 1 \) in our case and on left-hand side of (C2c) the first
term is ascending flow and the second term is channel flow.
With that we can combine solutions for horizontal flow far off
the mount axis (Couette/Poiseuille flow) with solutions for
ascending flow below the mount [e.g., Hansen and Kelmanson,
1994]. Assuming a new local coordinate system \( x' = x, y' = -y - (\bar{h} + (\bar{h} - y)/2) \), the boundary conditions for
the flow ascending near the symmetry axis would be \( u = v = 0; \)
\( du/dy' = 0 \) at \( x' = 0, y' = 0 \) (beneath the mount axis).
As pointed out by Davies [1994], it is impossible to provide an
analytical or simplified semianalytical solution for the case
when the viscosity \( \mu \) is defined exactly through the power law
(7). We therefore assume that the viscosity \( \mu \) in the ascending
flow is constant and equal to \( \mu = \bar{\mu} (a_j) \), where \( \bar{\mu} (a_j) \) is
the depth-averaged value of the effective nonlinear viscosity
defined from the solution for the channel flow (Appendix B) at
distance \( x = a_j \). Use of constant viscosity is, however, not
a severe simplification, because \( a_j \) is small.

Introducing vorticity function \( \xi = \text{rot} v = (\partial u/\partial y) - (\partial v/\partial x) = \nabla^2 \psi \), assuming laminar flow, we then have [Talbot
and Jarvis, 1984; Fletcher, 1988]
\[
\mu \frac{\partial \xi}{\partial x} = \frac{\partial p}{\partial y}, \quad \mu \frac{\partial \xi}{\partial y} = -\frac{\partial p}{\partial x}, \quad \xi = \nabla^2 \psi \tag{C3}
\]

The upper surface of the fluid, streamline \( \psi = 0 \), is taken to
be free of shear stresses, which leads to following conditions:
\( p \cos 2\alpha = 2 \mu \bar{\psi}/\partial y \partial x; \)
\( p \sin 2\alpha = \mu (\partial^2 \psi/\partial x^2 - \partial^2 \psi/\partial y^2) \), where \( \alpha \) is the slope of the surface (positive downward).
The symmetry of the flow requires \( \psi(-x, y') = -\psi(x, y') \).

The general solution in dimensionless variables [Talbot and
Jarvis, 1984; Fletcher, 1988]: \( X = \bar{h} \max x^2; Y = \bar{h}(0) y^2; p = (\mu \bar{q}/\pi \bar{h} \max^2) p'; \)
\( \psi = (q/\pi) \psi' \), where \( \bar{h} \max \) is the maximum height of the free
surface, is
\[
\psi = \tan^{-1} X/Y + XY/(X^2 + Y^2) + \sum_{n=0}^{\infty} (-1)^n(n + 1) Y^{2n+2} [(2n + 2)!]^{-1/2} X^{2n+1}(X) + \sum_{n=0}^{\infty} (-1)^n (n + 1) Y^{2n} [(2n + 3)!!]^{-1/2} Y^{2n+1}(X) + G(X) \tag{C4a}
\]
\[
p = K - \lambda Y + 2(Y^2 - X^2) [(X^2 + Y^2)^2 + \sum_{n=0}^{\infty} (-1)^n X^{2n+1} [(2n + 3)!!]^{-1} Y^{2n+1}(X) - G(X) \tag{C4b}
\]
\[
G(X) = \int_{0}^{X} \gamma (s) \; ds. \tag{C4c}
\]
where \( f \) and \( \gamma \) are expressed in expansion series (\( f^{(l)} \), \( \gamma^{(l)} \) are their \( l \)-th derivatives), \( \lambda = \pi \bar{q} \bar{h} \max^3/\mu \bar{q}, K \) is constant; \( \gamma \) and \( f \)
are determined numerically because the expressions for \( \psi \) and
\( p \) are nonlinear and cannot be solved analytically. The calculation
of \( \psi \) and \( f \) is done on the assumption of small curvature
of the upper surface which allows linear approximation of \( \gamma \)
and \( f \), i.e., as \( \gamma = AX \) and \( f = BX \). Then the free surface can be
searched in the form of a parabolic function, e.g., \( \bar{h} \sim C -
D X^2 \) [Talbot and Jarvis, 1984].

Notation
\[
\begin{align*}
\tau_{xx}, \tau_{yy}, \tau_{xy} & \quad \text{viscous stress components, MPa.} \\
\sigma_{xx}, \sigma_{yy}, \sigma_{xy} & \quad \text{stress components, } \sigma = \tau - P I, \text{ MPa.} \\
p & \quad \text{pressure, MPa.} \\
v & \quad \text{velocity vector, mm/yr.} \\
u & \quad \text{horizontal velocity (x component of } v), \text{ mm/yr.} \\
v & \quad \text{vertical velocity (y component of } v), \text{ mm/yr.} \\
\mu & \quad \text{effective viscosity (10}^{9} \text{ to } 10^{23} \text{ Pa s), Pa s.} \\
k & \quad \text{coefficient of mass diffusivity, m}^2/\text{yr.} \\
dh & \quad \text{topographic uplift or subsidence, m.} \\
du & \quad \text{tectonic uplift (>0) or subsidence (<0), m.} \\
dc & \quad \text{denudation (<0) or sedimentation (>) m, km.} \\
\psi & \quad \text{stream function, } u = -\partial \psi/\partial y, \text{ and } v = -\partial \psi/\partial x, \text{ m}^2/\text{sec.} \\
\xi & \quad \text{vorticity function } \partial u/\partial y - \partial v/\partial x = \Delta \psi, \text{ s}^{-1}. \\
e & \quad \text{average strain rate } (k = \partial^2 \psi/\partial x^2 - \partial^2 \psi/\partial y^2), \text{ s}^{-1}. \\
f & \quad \text{flux in the ducile crust, m}^2/\text{sec.} \\
q & \quad \text{erosional flux per unit length, (m}^2/\text{sec})/\text{m.} \\
E & \quad \text{Young's modulus, } 8 \times 10^{10} \text{ N/m}^2. \\
\nu & \quad \text{Poisson's ratio, equal to 0.25.} \\
\lambda, \mu_e & \quad \text{Lamé's constants, N/m}^2. \\
A^* & \quad \text{material constant (power law rheology), Pa}^{-n} \text{ s}^{-1}. \\
n & \quad \text{stress exponent (power law rheology), equal to 3 to 5.} \\
H^* & \quad \text{activation enthalpy (power law rheology), } \text{kJ mol}^{-1}. \\
R & \quad \text{gas constant, equal to 8.314 J/mol K.} \\
T & \quad \text{temperature, } ^{ø}C, \text{ K.} \\
\gamma(y) & \quad \text{depth gradient of yield stress } \gamma(y) \sim d\sigma(e)/dy, \text{ Pa/m.} \\
w & \quad \text{approximate deflection of mantle lithosphere, m.} \\
\bar{M}_t & \quad \text{flexural moment per unit length, Nm/m.} \\
\bar{T}_s & \quad \text{longitudinal force per unit length, N.} \\
\bar{Q}_s & \quad \text{shearing force per unit length, N/m.} \\
p^* & \quad \text{surface load, Pa, N/m}^2.
\end{align*}
\]
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References


