Bayesian modeling of the under constrained 7.8 Tohoku-Oki aftershock
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Abstract
We used a fully Bayesian probabilistic formalism requiring no a priori spatial regularization to invert the slip distribution of the 7.8 Tohoku-Oki aftershock from the very dense Japan GPS network. We extracted the static surface displacement from 5 min high rate GPS times series and we considered a one dimensional elastic structure. This inverse problem is highly under constrained. We show here the slip distribution obtained using different diagnostic parameters such as the probability density function (pdf) of each parameter or a posteriori covariance.

Introduction
The 9.0 Tohoku-Oki earthquake, occurred on March 11, 2011, was the best ever recorded event of this magnitude. Here we focus on the magnitude 7.8 aftershock happened 28 minutes after the main event. This event is particularly interesting because of its location; historically many quakes happened near the coast of Japan at the boundary between the Pacific and the Eurasian plates but none of them near the latitude of 36 degrees (see figure 1). This aftershock allows us to investigate a poorly known area where the deficit of seismic and inter seismie deformation could lead to a possible future large event. We used high rate GPS data from the Geodetic Survey of Japan (GSJ) and a fully Bayesian probabilistic formalism requiring no a priori spatial regularization to model the slip distribution along the fault responsible for this aftershock. Even though the data network is particularly dense, its spatial configuration makes the inverse problem highly under constrained. This aftershock modeling was an opportunity to test how much information we can obtain from such a problem with a Bayesian approach.

A Bayesian probabilistic formalism
We used a code named CATMIP, for CASCading Adaptive Temporal Metropolitan In Parelto (20), to explore the range of likely fault slip models. Instead of inverting the matrix of the Green’s function, the code builds a histogram of possible slip model that fit the data the best taking into account the error on the data (well known) and the error on the Green’s functions (unknown, estimated by the code). At the end, we obtain a probability density function (pdf) of each fault patch parameters, for the strike slip and the dip slip component.

\[
\begin{align*}
    P(s & =i | r) = P(r | s =i)P(s =i) \\
    P(d & =j | r) = P(r | d =j)P(d =j)
\end{align*}
\]

Equation 1

The equation governing the algorithm is the Bayes theorem (1). Then, by defining the data, the slip model, G the Green’s functions and C the covariance on the data, we derive (2) (considering a Gaussian distribution of the likelihood of d and a uniform prior model E−φ).

Data

Figure 2: horizontal (green) and vertical (blue) data and their error ellipses derived from 5 min high rate GPS time series.

We derived the static displacement (shown in figure 2) at the surface from 5 min high rate time series. Since the aftershock occurred only 28 min after the main event, there is possibly some long period signal remained. Even if this network is very dense, its configuration is far from optimal. The entire network is west of the aftershock. So, the inversion is "just blind"; the resolution decreases with the longitude. This configuration makes the problem highly under constrained.

Slip model

Figure 3: slip model associated with the 7.8 2011/03/11 Tohoku-Oki aftershock and data fit. The blue (top) and green (bottom) lines are the predicted data, the red line is the model. The model assumes the layered media shown in figure 5.

We considered a curved fault geometry interpolating the regional seismic activity over a surface. The strike-slip and dip-slip components of 21 patches were inverted from the static data, 4 lines of 5 patches (22X23 km) and one big patch (22X15 km) near the trench for covariance issues. The amplitude and the direction of the slip on each patch are plotted in figure 4. The slip amplitude values are the means of the whole set of model tests by the algorithm.

Elastic structure

Figure 4: cross section of the model along the strike (left) and dip (right) directions. The expected slip amplitude values are the means of the whole set of model tests by the algorithm.

The expected values are the means of the whole set of model tests by the algorithm. The patch to patch variations are important for the fault geometry.

The residual show a low amplitude incoherent signal (see figure 5). However, the maximum slip is found in an area where the resolution is presumably low, so the uncertainties are important and hard to estimate.

Model error estimation

At some point, the change on the amplitude of the slip model needed to better fit the data is too important compare to the gain on the fit itself. We can estimate a critical value for each model and get a pdf of the estimated model error (see figure 6).

We see that the model error on the vertical data is huge compared to the horizontal. In response to this estimation, the algorithm doesn’t try to fit the vertical data as well as the horizontal and so the vertical resolution is very coarse.

Likelihoods and Covariances of the model parameters

Figure 5: posterior pdfs of the slip model components. The abscissa is the amplitude of the slip component, the ordinate is the corresponding normal distribution.

Since we do not use any a priori spatial regularization, we cannot calculate a resolution from usual methods. However, the information contained in the posterior pdfs of each parameter (shown in figure 7) gives us a lot of information. The fully resolved case would be a Dirac function. As we start form a prior uniform model with a Gaussian noise model, we expect Gaussian pdfs with a mean and a standard deviation. The mean is the best fitting value over the explored model space and the standard deviation tells us about the resolution and the covariances. We observed Dirichlet shapes (familiarity with a bivariate normal) for the best resolved patches, e.g., the model error on the vertical data is much smaller than on the horizontal data.

Conclusion

We found a static slip distribution of the 7.8 Tohoku-Oki aftershock using GPS data and a Bayesian probabilistic formalism with no a priori spatial regularization in a layered earth. We tried to find a metric to discriminate what we can resolve and what we cannot using the information contained in the pdfs. Their shapes, their standard deviations and their covariances seem to give coherent information. Further work will be necessary in order to define an absolute metric of the true information content on a Bayesian frame.

References